1. The curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^{\,2} - 2x_n^{\,3})$$

is used to find an approximate value for α.

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

2. A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{7x\mathrm{e}^x}{\sqrt{\mathrm{e}^{3x} - 2}} \qquad x > \ln \sqrt[3]{2}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{7\mathrm{e}^x(\mathrm{e}^{3x}(2 - x) + Ax + B)}{2(\mathrm{e}^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2\mathrm{e}^{3x} - 4}{\mathrm{e}^{3x} + 4}$$

(2)

The equation $x = \dfrac{2\mathrm{e}^{3x} - 4}{\mathrm{e}^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2\mathrm{e}^{3x_n} - 4}{\mathrm{e}^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2\mathrm{e}^{3x} - 4}{\mathrm{e}^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on next page,

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

Question 2 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

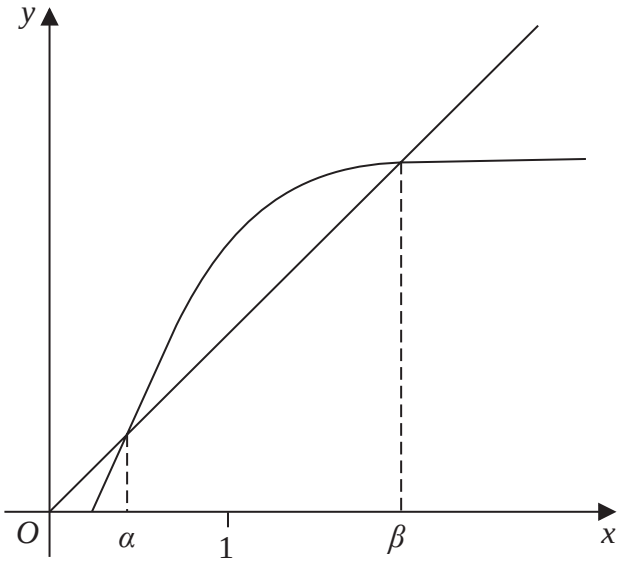
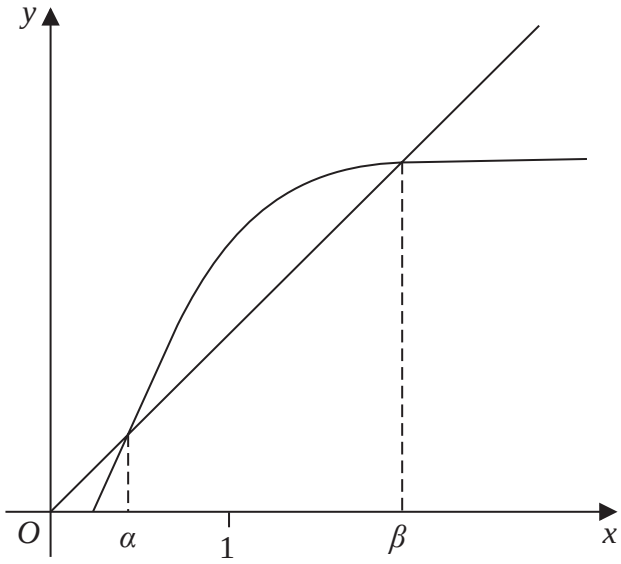


Diagram 1



copy of Diagram 1